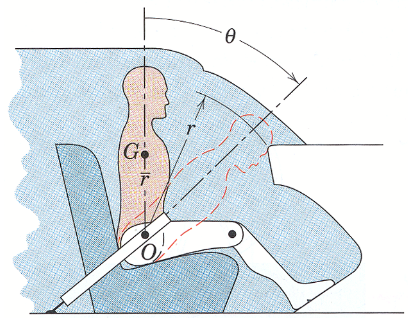
Review Problems - Final Exam



2. In a study of head injury against the dashboard of a car during sudden or crash stops where lap belts without shoulder straps are used, the segmented human model shown in the figure is analyzed. The hip joint O is assumed to remain fixed relative to the car and the torso above the hip is treated as a rigid body of mass *m* pinned at O. The center of mass of the torso is at G with the initial position of OG taken as vertical. The radius of gyration about O is k_O. If the car is brought to a stop with a constant deceleration *a*, determine the velocity *v* relative to the car with which the model's head strikes the dashboard. Substitute the values:

$$m = 50\ kg \qquad \bar{r} = 450\ mm \qquad r = 800\ mm \qquad k_O = 550\ mm \qquad \theta = 45^\circ \qquad a = 10g$$

(taken from Dynamics by Meriam and Kraige, Fourth Edition)

Hint: $I_O = I_G + m\bar{r}^2$

ans: $v = 11.73\ m/s$

Strategy: Since the car is moving at a non-constant velocity, we can't use COE to obtain a solution. We'll therefore apply the rate form equations in the displaced position (right before the head hits the dashboard) and work backward.

unk	eqs
v_H	1
W	2
a_{Gx}	3
a_{Gy}	4
α_{GO}	5
ω_{GO}	6
a_{Ox}	7
a_{Oy}	8

Kinetics:

COAM(RF) at O

$$-W\bar{r}\sin\theta = -I_G\alpha_{GO} + ma_{G_y}\bar{r}\sin\theta - ma_{G_x}\bar{r}\cos\theta \tag{1}$$

Kinematics:

$$\bar{a}_G = \bar{a}_O + \bar{a}_{G/O}$$
$$= \bar{a}_O + \bar{\alpha}_{GO} \times \bar{r}_{G/O} - \omega_{GO}^2 \bar{r}_{G/O}$$

$$\hat{i}: \quad a_{G_x} = a_{O_x} + \alpha_{GO} r_{G/O_y} - \omega_{GO}^2 r_{G/O_x}$$
$$\hat{j}: \quad a_{G_y} = a_{O_y} - \alpha_{GO} r_{G/O_x} - \omega_{GO}^2 r_{G/O_y} \tag{2,3}$$

Note the signs in (2,3) preceeding the angular acceleration terms. What has happened?

Some early kinematics reminds of the relationship between angular acceleration and angular velocity

$$a = w\frac{dw}{dq} \tag{4}$$

and once we know the angular velocity

$$v_H = w_{GO} r \tag{5}$$

Constraints and Geometry:

$$\bar{a}_O = -10g\hat{i} + 0\hat{j} \quad \Rightarrow \quad a_{O_x} = -10g,\ a_{O_y} = 0 \tag{6,7}$$

$$\bar{r}_{G/O} = \bar{r}\sin q\ \hat{i} + \bar{r}\cos q\ \hat{j} \quad \Rightarrow \quad r_{G/O_x} = \bar{r}\sin q,\ r_{G/O_y} = \bar{r}\cos q$$

Other:

$$W = mg \tag{8}$$

Solving:

Substituting (2,3,6,7,8) into (1) along with the position vector components and rearranging gives:

$$a_{GO} = \frac{mg\bar{r}}{I_G + m\bar{r}^2}\left(\sin q + 10\cos q\right)$$

What happened to the angular velocity?

Substituting the hint and relating the mass moment of inertia to the radius of gyration

$$a_{GO} = \frac{g\bar{r}}{k_o^2}\left(\sin q + 10\cos q\right)$$

Substituting into (4) and integrating

$$\int_0^{w_{GO}} w\,dw = \int_0^{q} a_{GO}\,dq$$

gives

$$w_{GO} = 14.66\ rad/s$$

which, when substituted into (5)

$$v_H = 11.73\ m/s \quad (26\ mph)$$