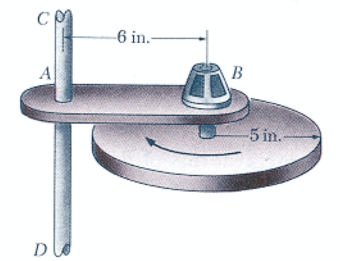
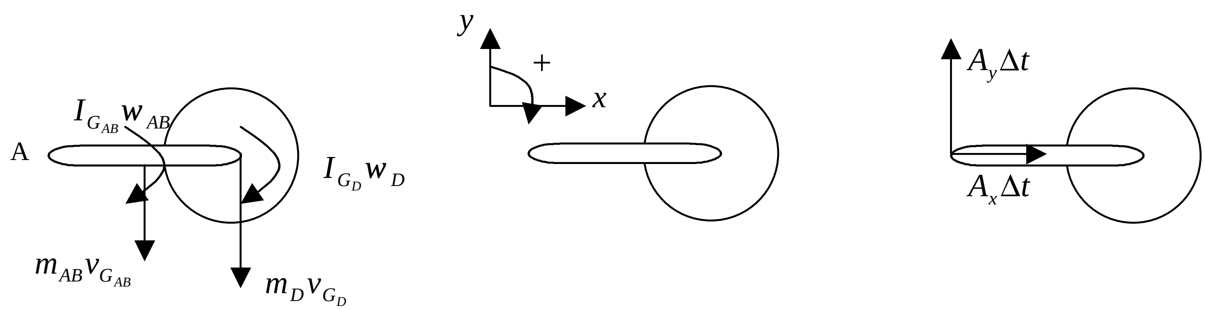
Example Problem - Le 14

17.95 A 10-lb disk is attached to the shaft of a motor mounted on arm AB which is free to rotate about the vertical axle CD. The arm-and-motor unit has a moment of inertia of 0.032 lb ft s^2 with respect to the axle CD, and the normal operating speed of the motor is 360 rpm. Knowing that the system is initially at rest, determine the angular velocities of the arm and of the disk when the motor reaches a speed of 360 rpm.
(*taken from Vector Mechanics for Engineers, 5th Edition by Beer & Johnston*)



Strategy: Use COAM(FT) (*impulse-momentum*)

Known: $W_D = 10\,lb \qquad I_{A_{AB}} = 0.032\,lb\ ft\,s^2 \qquad w_m = 360\,rpm = 37.7\,rad/s$



After - Before = During

Kinetics: COAM(FT) about A

$$I_{G_{AB}} w_{AB} + I_{G_D} w_D + m_{AB} V_{G_{AB}} r_{G_{AB}/A} + m_D v_{G_D} r_{G_D/A} - 0 = 0 \qquad (1)$$

Kinematics:

relate v_G and ω

$$v_{G_{AB}} = w_{AB} r_{G_{AB}/A} \qquad (2)$$

$$v_{G_D} = w_D r_{G_D/A} \qquad (3)$$

relative angular velocity

$$w_D = w_{AB} + w_{D/AB} = w_{AB} + w_m \qquad (4)$$

Other:

$$I_{A_{AB}} = I_{G_{AB}} + m_{AB} r_{G_{AB}/A}^2 \qquad I_{G_D} = \frac{1}{2} m_D r_D^2 \qquad (5)$$

Substituting (2) and (3) into (1):

$$I_{G_{AB}} w_{AB} + I_{G_D} w_D + m_{AB} w_{AB} r_{G_{AB}/A}^2 + m_D w_D r_{G_D/A}^2 = 0$$

$$w_{AB}\left(I_{G_{AB}} + m_{AB} r_{G_{AB}/A}^2\right) + w_D\left(I_{G_D} + m_D r_{G_D/A}^2\right) = 0 \qquad (6)$$

Substituting (4) and (5) into (6):

$$w_{AB} I_{A_{AB}} + \left(w_{AB} + w_m\right)\left(\frac{1}{2} m_D r_D^2 + m_D r_{G_D/A}^2\right) = 0$$

$$w_{AB} = \frac{w_m \left(\frac{1}{2} m_D r_D^2 + m_D r_{G_D/A}^2\right)}{I_{A_{AB}} + \frac{1}{2} m_D r_D^2 + m_D r_{G_D/A}^2} \tag{7}$$

Solving (7):

$$\boxed{\begin{array}{l} w_{AB} = 71.1\hat{k}\ rpm \\ w_D = -289\hat{k}\ rpm \end{array}}$$