Conservation and Accounting Review

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In ES201 the basic idea of conservation and accounting was investigated and you were introduced to conservation of mass, charge, linear momentum, angular momentum, and energy and an accounting equation for entropy. In ES204 we will investigate the application of these principles to mechanical systems. In this class we will be primarily using conservation of linear momentum, conservation of angular momentum and conservation of energy. The rate form of these equations are shown below.

Linear Momentum:

$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F} + \sum_{in} \dot{m}_i \vec{v}_i - \sum_{out} \dot{m}_o \vec{v}_o \qquad (1)$$

where for a system consisting of a single object $\vec{P}_{sys} = m\vec{v}_G$ or for a system of several objects $\vec{P}_{sys} = \sum_{i=1}^{n} (m\vec{v}_G)_i$

Angular Momentum about point O:

$$\frac{d\vec{L}_{sys_0}}{dt} = \sum \vec{M}_o + \sum_{in} \vec{r} \times \dot{m}_i \vec{v}_i - \sum_{out} \vec{r} \times \dot{m}_o \vec{v}_o \qquad (2)$$

where for the plane motion of a single rigid body $\vec{L}_{sys_0} = I_G \vec{\omega} + \vec{r} \times m\vec{v}_G$

Energy:

$$\frac{dE_{sys}}{dt} = \dot{Q} + \dot{W} + \sum_{in} \dot{m}_i \left(h + \frac{v^2}{2} + gz \right)_i - \sum_{out} \dot{m}_o \left(h + \frac{v^2}{2} + gz \right)_o \qquad (3)$$

where $E_{sys} = E_k + E_G + E_s + U$

In this class we will primarily be looking at closed systems so Eqs (1-3) become:

Linear Momentum:

$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F} \qquad (4)$$

Angular Momentum:

$$\frac{d\vec{L}_{sys_0}}{dt} = \sum \vec{M}_o \qquad (5)$$

Energy:

$$\frac{dE_{sys}}{dt} = \dot{W} \qquad (6)$$

In most dynamics books the use of Eqs. (4-5) are referred to as direct applications of Newton's 2nd Law (Chapters 12 and 16 in Beer aned Johnson), the use of the finite time forms of Eqs. (4-5) are referred to as impulse momentum methods (Chapters 13 and 17 in Beer aned Johnson) and the use of the finite time form of Eq. (6) is referred to as the Work-Energy method (Chapters 13 and 17 in Beer aned Johnson).

A summary of the equations you are expected to know from ES201 and how they are typically referred to in the text is shown below.

Principle	ES201 Name	Dynamics Name	Comments
$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F}$ $\frac{d\vec{L}_{sys_0}}{dt} = \sum \vec{M}_o$	Rate form for conservation of linear and angular momentum for a closed system.	Direct application of Newton's Laws	When to use: • want to find forces and/or accelerations • want to find velocities and/or distance traveled (which can be found by separating variables and integrating the basic kinematic relationships) Other: • Be careful! These are vector equations. • The book uses H_0 for angular momentum instead of L_0.
$\Delta\vec{P}_{sys} = \int_{t_1}^{t_2} \vec{F}dt$ $\Delta\vec{L}_{sys_0} = \int_{t_1}^{t_2} \vec{M}_0 dt$ or if there are impulsive loads acting on the system $\Delta\vec{P}_{sys} = \sum \vec{F}_i \Delta t$ $\Delta\vec{L}_{sys_0} = \sum (\vec{M}_0)_i \Delta t$ where F_i and M_i are only external impulsive forces and moments acting on the system.	Finite time form of conservation of linear and angular momentum for a closed system.	Impulse-momentum methods	When to use: • have an impact or impulsive forces • the system consists of several objects • given a force as a function of time • want to find velocities, times, or forces (especially impulsive forces) Other: • Be careful! These are vector equations. • The book uses H_0 for angular momentum instead of L_0.
$\Delta E_{sys} = W$	Finite time form of conservation of energy for an adiabatic closed system.	Work-energy methods.	When to use: • have two locations in space • given a force as a function of position • want to find velocities, distances, or forces (sometimes) Other: • This is a scalar equation

Recall that the key steps in applying a conservation principle are:
1. Identify System
2. Identify Property to be counted
3. Time Interval

Miscellaneous comments

The homework assignments from ES201 that are particularly relevant to this class are problems 13.1, 13.2, 14.1, 14.2, 14.3, 15.1, 15.2, 16.1, 16.2, 16.3, 17.1, 17.2, 17.3, 18.1, 25.1, 25.2, 26.1, 26.2, and 26.3. It is expected that you understand and are able to do all of these problems. If there are some of these that you cannot do please see your professor. The textbook for this course is *Vector Mechanics for Engineers: Dynamics* by Beer and Johnson. Most of the material in chapters 11-14 has already been covered in ES201. We will be filling in any deficiencies from these chapters in this course as well as adding material in the area of rigid body kinematics.

One of the primary purposes of this class is to deepen your understanding of how objects move as a result of forces acting on them. At most schools this material is presented in a course called Dynamics. The study of motion can be divided into two distinct parts: kinematics and kinetic. Kinematics is the study of what I like to call "the geometry of the motion." In other words, describing the motion without reference to the forces causing the motion. The second part, kinetics, relates the motion to the forces acting on the system. Most dynamics textbooks do not assume students have a good understanding of the basic kinetics principles prior to the class. For this reason they are typically organized so that kinematics is first presented followed by the kinetics. Frequently, however, the kinematics really lacks motivation since it is not clear how it will subsequently be used. Fortunately, you are already familiar with the basic kinetics principles! Because of this, we will be jumping around in the textbook so that the kinematics you are learning can immediately be used in conjunction with one or more of the conservation principles providing a clear motivation for the kinematics in addition to practice using the conservation principles.