Relative Motion

This is one of the most important topics in this course.

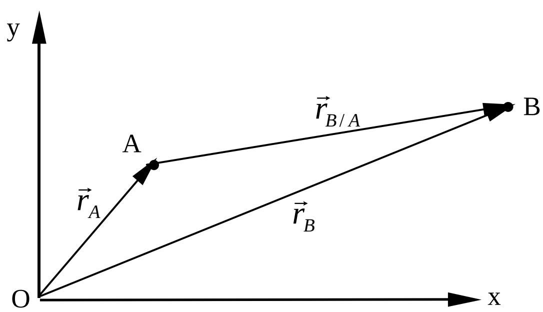
All motion is relative. We have to have some frame of reference (coordinate system) in order to measure position, velocity and acceleration.

For this course, these frames of reference can be put into two groups. First, reference points that are fixed, usually with respect to the earth or the laboratory. I know these frames of reference are moving, with respect to say the sun or the stars, but they don't have much acceleration to speak of. It's usually safe to think of them as "fixed."

The second type of reference is going to be discussed at length in these notes. It is a reference system that is itself moving.

Let's say that observer A is at the origin of a moving reference frame. He or she observes the motion of another point B. A could measure a position, velocity and acceleration for B. But these motions are not the same that an observer in the fixed frame would measure by observing B. The difference would be A's motion.

Here are the basic equations, which stem from the simple drawing.



O = origin of fixed coordinate system
A = origin of moving coordinate system
B = another moving point

Note how we find that to get to point B we can either go there straight ahead, or we can go first to A and then head over to B. This idea can be expressed vectorially as follows:

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \tag{1}$$

In words, Eq. (1) is: the position of B is equal to the position of A plus the relative position of B with respect to A. Now let's differentiate the previous equation with respect to time. What we obtain is a similar statement about the velocities.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \tag{2}$$

or in words: the velocity of B is equal to the velocity of A plus the relative velocity of B with respect to A. Now differentiate with respect to time again.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \tag{3}$$

This equation says the acceleration of A plus the acceleration of B as seen from A is equal to the acceleration of B measured in the same fixed frame that A's position is measured in.

For now, lets not let the moving frame rotate. This is a topic we will investigate in the future

Questions

The following questions are just to get you started thinking. It's important to be able to imagine that you are at point A, which is moving and observe the motion of another point B. The following questions deal with velocities, and are an important starting point.

1. You are heading east on US 40 in front of Rose-Hulman at 50 mph. Another car is heading east at 45 mph just ahead of you. What is the relative velocity of the car? Magnitude and direction, please.

2. You are heading east on US 40 in front of Rose-Hulman at 50 mph. Another car is heading west at 45 mph and approaching you. What is the relative velocity of the car? Please give magnitude and direction.

3. You are heading west on US40 approaching the light for Indiana 46. Your speed is 40 mph. A car is approaching the same light heading north on 46 at 30 mph. What is the velocity of this car, relative to your car? Give an approximate speed and direction.

4. One person, A, is riding on an escalator, ascending to the next floor. B is on the next floor walking so that he is directly above A at all times. What is the direction of B's velocity as seen from A? What is the direction of A's velocity as seen by B?

5. A car is racing east along a level ground at 80 mph. It is moving straight and never turning. What is the velocity of a telephone pole with respect to the driver of the car?

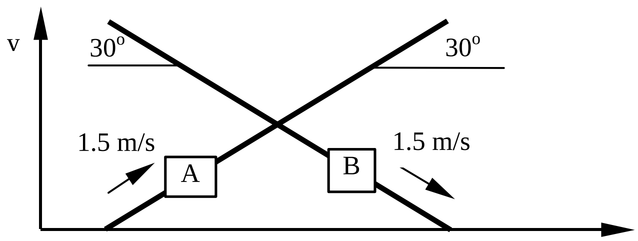
6. In the car just referred to what is the velocity of a point at the very center of the right front wheel (right on the centerline of the axle) with respect to the driver? (This is an easy one.)

7. What is the velocity of the point on the tire that is in contact with the ground with respect to the driver (for simplicity, assume the tire is perfectly round and that the car is not skidding). What is the velocity of this point with respect to an outside, stationary observer?

8. What is the velocity of the point on the tire that is at the top of the tire, that is the 12 o'clock position relative to the driver? Relative to the stationary outside observer?

Example:

Known: There are two escalators, as shown in the figure.
Find: Determine the relative velocity of B with respect to A.
Given: A is ascending at a constant 1.5 m/s and B is descending at a constant 1.5 m/s.

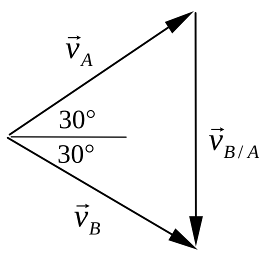


Analysis :
Strategy: Use relative motion equations

We know $$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

We have two approaches at this point, 1) vector diagram and 2) vector algebra.

Vector diagram approach:



$\vec{v}_A$ =1.5 m/s

$\vec{v}_B$ =1.5 m/s

Clearly from the diagram and the angles given in the problem this is an equilateral triangle so $\vec{v}_{B/A} = -1.5\hat{i}$

Vector algebra approach: Write all the velocity in terms of their components and then equate components.

$$\vec{v}_B = 1.5\cos(30)\hat{i} - 1.5\sin(30)\hat{j}, \; \vec{v}_A = 1.5\cos(30)\hat{i} + 1.5\sin(30)\hat{j}, \; \vec{v}_{B/A} = v_{B/A_x}\hat{i} + v_{B/Ay}\hat{j}$$

substituting into the relative velocity equation we get:

$$1.5\cos(30)\hat{i} - 1.5\sin(30)\hat{j} = 1.5\cos(30)\hat{i} + 1.5\sin(30)\hat{j} + v_{B/A_x}\hat{i} + v_{B/Ay}\hat{j}$$

equating components gives:

$$\hat{i}: \quad 1.5\cos(30) = 1.5\cos(30) + v_{B/A_x} \longrightarrow v_{B/A_x} = 0$$

$$\hat{j}: \quad -1.5\sin(30) = 1.5\sin(30) + v_{B/Ay} \longrightarrow v_{B/Ay} = -1.5$$

See also examples 2.15 and 2.16 in the text.