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Finite-Time Form of Linear and Angular Momentum

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In the text the finite-time forms of linear and angular momentum are referred to as "impulse-momentum methods". For a closed system these can be written as:

$$\Delta\vec{P}_{sys} = \int_{t_1}^{t_2} \vec{F}dt \qquad \text{and} \qquad \Delta\vec{L}_{sys_0} = \int_{t_1}^{t_2} \vec{M}_0 dt \qquad (1), (2)$$

If there are any impulsive forces (an impulsive force is a large force that acts over a small time) acting on the system then non-impulsive forces (weight, springs, etc.) can be neglected and Eq. 1-2 become

$$\Delta\vec{P}_{sys} = \sum \vec{F}_i \Delta t \qquad \text{and} \qquad \Delta\vec{L}_{sys_0} = \sum (\vec{M}_0)_i \Delta t \qquad (3), (4)$$

where $\vec{F}_i$ and $\vec{M}_i$ are external impulsive forces acting on the system. Recall for a rigid body the linear and angular momentum are

$$\vec{P}_{sys} = m\vec{v}_G \qquad \text{and} \qquad \vec{L}_{sys_O} = I_G\vec{\omega} + \vec{r}_{G/O} \times m\vec{v}_G \qquad (5), (6)$$

When to use

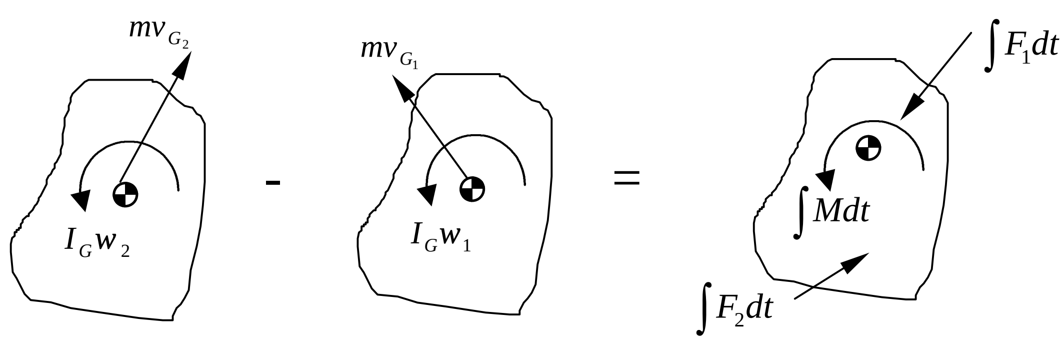
The finite time form of conservation of linear and angular momentum are typically used when:

- there is an impact or impulsive forces in the problem
- there are several interacting objects
- there is a force as a function of time
- want to find velocities, times or forces (especially impulsive forces)

Procedure

Since Eqs (1-2) and Eqs. (3-4) are vector equations it is often useful to draw impulse momentum diagrams as shown below.

Picture :



In Words :	System momentum after the time interval	-	System momentum before the time interval	=	Impulsive forces acting during the time interval
Equations :	$\vec{P}_{sys_2}$	-	$\vec{P}_{sys_1}$	=	$\sum \vec{F}_i \Delta t$
	$\left(\vec{L}_{sys_O}\right)_2$	-	$\left(\vec{L}_{sys_O}\right)_1$	=	$\sum \left(\vec{M}_O\right)_i \Delta t$

We therefore have three scalar equations

1. Linear momentum in the x-direction
2. Linear momentum in the y-direction
3. Angular momentum (moment of the momentum) about any axis