Supplementary Homework Problem (HW Set 22)

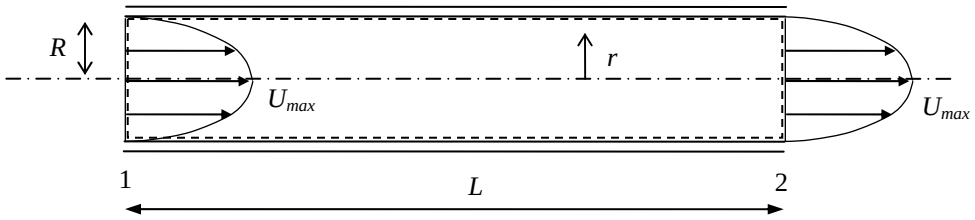
Due at the beginning of Lecture 24

Friction factor of a laminar flow in a circular pipe

The velocity profile in the fully developed region of a circular pipe is invariant with streamwise position. In the laminar regime, it can be described analytically by

$$u(r) = U_{\max}\left(1 - \frac{r^2}{R^2}\right)$$

where r is the radial distance measured from the center of the pipe.



The friction factor for a horizontal pipe is defined by

$$P_1 - P_2 = f\,\frac{L}{D}\,\frac{\rho U_{avg}^2}{2}.$$

Apply conservation of linear momentum to the chosen system (bounded by dashed lines) and show the following well-known relation

$$f = \frac{64}{\mathrm{Re}_D}$$

holds in the laminar flow regime.