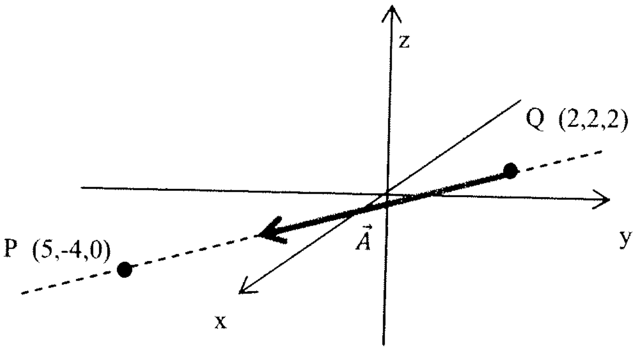
Problem 1 – Short Answer -- *35 points*

(a) Point Q has coordinates (2,2,2). A force, $\vec{A}$, whose magnitude is 70 N, acts at Q along a line toward Point P (5, -4, 0).



i. Write force $\vec{A}$ in Cartesian vector form.

5 points

$\vec{QP} = (5\vec{i} - 4\vec{j} + 0\vec{k}) - (2\vec{i} + 2\vec{j} + 2\vec{k})$
$= 3\vec{i} - 6\vec{j} - 2\vec{k}$] 1 point

$\vec{\lambda}_{QP} = \dfrac{3\vec{i} - 6\vec{j} - 2\vec{k}}{\sqrt{3^2 + 6^2 + 2^2}} = \frac{3}{7}\vec{i} - \frac{6}{7}\vec{j} - \frac{2}{7}\vec{k}$] ÷ by mag 2 points

$\vec{A} = 70\,N \cdot \vec{\lambda}_{QP} = \underline{(30\vec{i} - 60\vec{j} - 20\vec{k})\,N}$] mult by 70 N 2 points

sign error −1 UNITS −1
direction reversed −1

ii. What angle does force $\vec{A}$ make with the x-axis? Give your answer in degrees.

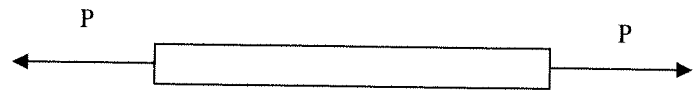
5 points

$\theta_x = \cos^{-1}\left(\frac{3}{7}\right) = \underline{1.128 \text{ rad} = 64.6^\circ}$

work in 2D using arctan — incorrect answer −5
used arctan (3/7) −2
used arccos $\left(\frac{3}{7 \cdot 7}\right)$ −2
used arccos (3/7) wrong angle −1

(c) An aluminum bar has the following specifications:

Length	10 in
Diameter	0.505 in
Cross sectional area	0.2 in^2
Modulus of Elasticity	1.0 x 10^7 lb/in^2
Yield Strength	38,000 lb/in^2
Ultimate Tensile Strength	65,000 lb/in^2
Coefficient of Thermal Expansion	12.5 x 10^{-6}/°F



5 points

i. If the bar carries an axial force of 3100 lb, calculate the axial strain.

(3) $\sigma = \frac{3100}{0.2} \frac{lb}{in^2} = 15{,}500 \frac{lb}{in^2}$ (2) $\epsilon = \frac{\sigma}{E} = 0.00155 = 1.55 \times 10^{-3}$

• σ ok no strain −2 • did $\frac{3100}{E}$ −3 • gave units (MPa) −1 • numerical error −1

5 points

ii. Calculate the smallest force which would leave a noticeable plastic (permanent) deformation after unloading. most others −5

$F_y = \sigma_{ys} \cdot A = 7600$ lb

• units −1 • just gave YS −3 • other errors −5

5 points

iii. What is the smallest applied force which would cause the bar to break?

$F_{max} = \sigma_{UTS} \cdot A = 13000$ lb

• units −1 • just gave UTS −3 • other errors −5

5 points

iv. If we heat the bar up by 50°F, what is the change in length of the bar?

$\delta_{th} = \alpha \Delta T L = (12.5 \times 10^{-6}/°F)(50°F)(10 \text{ in}) = 0.00625$ in

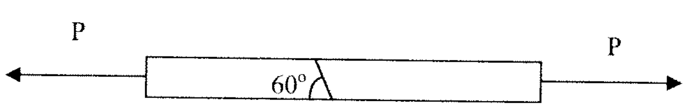
I also accepted $\delta_{th} + 0.00155(10) = 0.0218$ in assuming load from i

• UNITS −1

(d) A 1/8" diameter wooden dowel is sliced at a 60° angle, and then it is joined with glue. What is the normal stress on the glued plane due to an applied force of 100 pounds?

5 points

$\theta = 60° \Rightarrow 2037$ psi
$\theta = 30°$



$\sigma = \frac{P}{A} \cos^2\theta$ $A = \frac{\pi}{4}\left(\frac{1}{8}\right)^2 = 0.01227 \text{ in}^2$

$= \left[\frac{100 \text{ lb}}{0.01227 \text{ in}^2}\right] \cos^2(30°) = 8149.7 \cos^2(30°) = 6110 \frac{lb}{in^2}$

• wrong area −1 • wrong θ −2 too many digits −1

• units −1 • used FBD approach, got it wrong −3 or −4

(c) An aluminum bar has the following specifications:

Length	10 in
Diameter	0.505 in
Cross sectional area	0.2 in^2
Modulus of Elasticity	1.0×10^7 lb/in^2
Yield Strength	38,000 lb/in^2
Ultimate Tensile Strength	65,000 lb/in^2
Coefficient of Thermal Expansion	$12.5 \times 10^{-6}/°F$

i. If the bar carries an axial force of 3100 lb, calculate the axial strain.

ii. Calculate the smallest force which would leave a noticeable plastic (permanent) deformation after unloading.

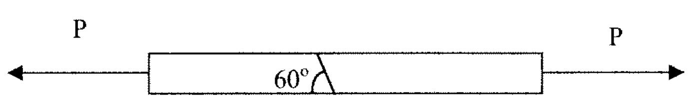
iii. What is the smallest applied force which would cause the bar to break?

iv. If we heat the bar up by 50°F, what is the change in length of the bar?

4 pts
units 1 pt

$$\delta = \alpha \Delta T L = (12.5 \times 10^{-6}/°F)(50°F)(10")$$
$$= .00625 \text{ in}$$

(d) A 1/8" diameter wooden dowel is sliced at a 60° angle, and then it is joined with glue. What is the normal stress on the glued plane due to an applied force of 100 pounds?



$$\sigma = \frac{P}{A}\cos^2(30) = \frac{100 \text{ lbs}}{\pi (1/8")^2/4}\cos^2(30) = 6112 \text{ psi}$$

area $\pi (1/8")^2/4 = .01227 \text{ in}^2$

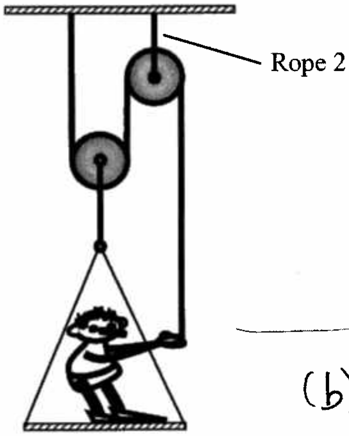
using $\pi (1/8")^2$ for area −1
other wrong areas −2

using 60° instead of 30° −2
units −1

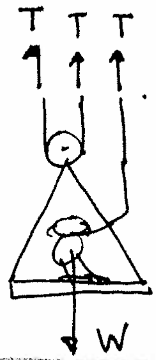
Problem 2 – *30 points*

A window washer uses a system of frictionless and massless pulleys to hoist himself up the side of a building. The ropes in the pulley system are light and inextensible. The window washer has a weight of W and the platform on which he stands has negligible weight.

(a) Find the force with which the window washer pulls on the rope in terms of the weight W.
(b) Find the tension in Rope 2 in terms of the weight W.
(c) Find the force the window washer's feet exert on the platform in terms of the weight W.



(a)



$\uparrow \sum F_y = 0$

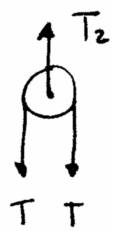
$+3T - W = 0$

$T = \frac{W}{3}$ ◀

FBD 6 PTS
EQUIL + SOLVE 5 PTS

11 PTS

(b)



$\uparrow \sum F_y = 0$

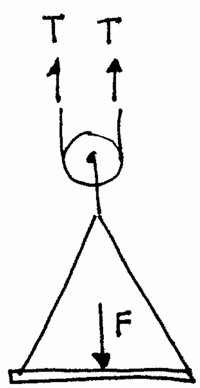
$T_2 - 2T = 0$

$T_2 = 2T = 2\left(\frac{W}{3}\right) = \frac{2}{3}W$ ◀

FBD 4 PTS
EQUIL + SOLVE 5 PTS

9 PTS

(c)



$\uparrow \sum F_y = 0$

$2T - F = 0$

$F = 2T = 2\left(\frac{W}{3}\right) = \frac{2}{3}W$ ◀

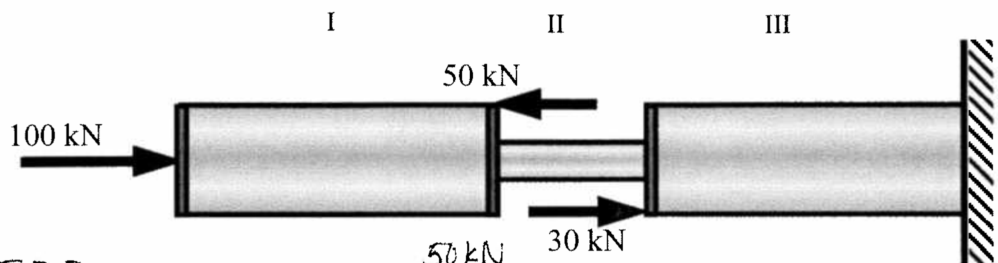
FBD 5 PTS
EQUIL + SOLVE 5 PTS

10 PTS

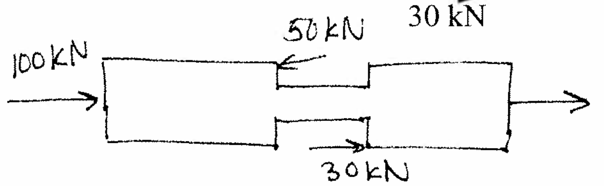
Problem 3 – *35 points*

Three cylindrical bars (I, II, and III) are attached to each other and to a wall at one end, and are loaded as shown below. Sections I and III are hollow with an inner diameter of 20 mm and an outer diameter of 30 mm. Section II has a solid, circular cross-section of an unknown diameter.

(a) Find the reaction force at the wall.
(b) Determine the average normal stress in section III.
(c) If the stress in section II cannot exceed 50 MPa, determine the minimum diameter of the solid cylinder in section II.



10pts (a) FBD:



(5pts FBD)

$\Sigma F_x = 0 = 100 - 50 + 30 + R \rightarrow$ $\boxed{R = -80\ kN}$ (a) (5pts eq + correct ans. w/ units)

10pts (b) FBD: cut section III (3pts FBD) (2pts Equation + Force)

$\Sigma F_x = 0 \rightarrow -F_3 + R = 0 \rightarrow R = F_3$

$F_3 = -80\,kN$ (compression)

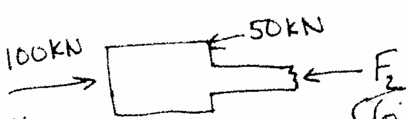
$\sigma_3 = \frac{F_3}{A_3}$ $\quad A_3 = \pi\left(\left(\frac{d_o}{2}\right)^2 - \left(\frac{d_i}{2}\right)^2\right) = \pi\left(15\,mm^2 - 10\,mm^2\right) = 392.7\ mm^2$ (2pts → Area)

$= 0.0003927\ m^2$

$\sigma_3 = \frac{-80000\ N}{0.0003927\ m^2} = \boxed{-203.7\ MPa}$ (− implies compressive σ)

(b) (3pts σ w/units + correct sign)

15pts (c) FBD: cut section II (2pts force)



(6pts FBD)

$\Sigma F_x = 0 = 100 - 50 - F_2 \rightarrow F_2 = 50$ (compression)

$\sigma_2 = \frac{F_2}{A_2}$ $\quad A_2 = \pi\left(\frac{d}{2}\right)^2$ (2pts area)

(3pts σ_{max} + d relationship)

$\sigma_2 = \sigma_{max} = \frac{F_2}{\frac{\pi}{4}d^2} \rightarrow d^2 = \frac{4F_2}{\pi\sigma_{max}} \rightarrow d = \sqrt{\frac{4F_2}{\pi\sigma_{max}}}$

(2pts final ans w/units)

$d = \sqrt{\frac{4(50)\,kN}{\pi(50000\,kPa)}} = 0.0357\ m$ $\quad \boxed{d = 35.7\ mm}$ (c)