Summarized Results for 100,043 "Safe" Prime Zero Omitted

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General

All: 100,041 Graphs
Primitive Root: 50,020 Graphs
Not Primitive Root: 50,021 Graphs

Largest Cycle: 100,042 ($g = 20,812$ and $94,034$)
Longest Tail: 1,448 ($g = 89,339$)
Shortest Maximum Cycle: 1 ($g = 72,116$ and $91,980$ and $95,997$)

Number of Components

Observed All:

$$923,855/100,041 \approx 9.23$$

Maps

Theoretical:

$$\frac{1}{2}\log n = \frac{1}{2}\log 100,043 \approx 5.76$$

Observed Not PR:

$$319,587/50,021 \approx 6.39$$

Permutations

Theoretical:

$$\sum_{i=1}^{100,043} \frac{1}{i} \approx 12.09$$

Observed PR:

$$604,268/50,020 \approx 12.08$$

Number of Cyclic Nodes

Observed All:

$$5,023,873,923/100,041 \approx 50,218.15$$

Maps

Theoretical:

$$\sqrt{\pi n/2} = \sqrt{\pi 100,043/2} \approx 396.418$$

Observed Not PR:

$$19,773,083/50,021 \approx 395.296$$

Permutations

Theoretical:

$$100,042$$

Observed PR:

$$5,004,100,840/50,020 = 100,042$$

Number of Tail Nodes

Observed All:

$$4,984,427,799/100,041 \approx 49,823.85$$

Maps

Theoretical:

$$n - \sqrt{\pi n/2} = 100,042 - \sqrt{\pi 100,042/2} \approx 99,645.58$$

Observed Not PR:

$$4,984,427,799/50,021 \approx 99,646.70$$

Permutations

Theoretical:

$$0$$

Observed PR:

$$0/50,020 = 0$$

Number of Terminal Nodes

Observed All:

$$2,502,150,460/100,041 \approx 25,011.25$$

Maps

Theoretical:

$$e^{-1}n = e^{-1} * 100,042 \approx 36,803.395$$

Observed Not PR:

$$2,502,150,460/50,021 = 50,021.99996$$

All values of g except $100,042$ that were not PR had exactly $50,022$. $g = 100,042$ had a total of $100,041$. Since $100,042 \equiv -1 \mod n$ this exception makes sense. In the other cases, obviously, $g^0 \equiv g^{p-1} \equiv 1 \mod n$.

Permutations

Theoretical:

$$0$$

Observed PR:

$$0/50,020 = 0$$

Number of Image Nodes

Observed All:

$$100,043 - 25,012.25 \approx 75,030.75$$

Maps

Theoretical:

$$(1 - e^{-1})n = (1 - e^{-1}) * 100,043 \approx 63,239.24$$

Observed Not PR:

$$100,043 - 50,022.99996 \approx 50,020.00$$

The notes under Number of Terminal Nodes also apply here since Terminal Nodes+Image Nodes = n.

Permutations

Theoretical:

$$n - 1 = 100,0437 - 1 = 100,042$$

Observed PR:

$$100,043 - 1 = 100,042$$

Average Tail Length

Observed All:

197.95

Maps

Theoretical:

$$\sqrt{\pi n/8} = \sqrt{\pi 100,042/8} \approx 198.21$$

Observed Not PR:

197.96

Have to double check with new run that does not include 0.

Permutations

Theoretical:

$$0$$

Observed PR:

$$0$$

Average Cycle Length

Observed All:

$$25,089.18$$

Maps

Theoretical:

$$\sqrt{\pi n/8} = \sqrt{\pi 100,043/8} \approx 198.21$$

Observed Not PR:

$$198.315$$

Permutations

Theoretical:

$$\frac{n+1}{2} = \frac{100,043+1}{2} = 50,022$$

Observed PR:

$$49,980.6$$

Maximum Cycle Length

Observed All:

$$31,321.1$$

Maps

Theoretical:

$$c_1\sqrt{n} \approx 0.78248\sqrt{100,043} \approx 247.495$$

Observed Not PR:

$$247.256$$

Permutations

Theoretical:

$$0.62432965n = 0.62432965 * 100043 \approx 62,495.81$$

Observed PR:

$$62,395.5$$

Maximum Tail Length

Observed All:

$$271.41$$

Maps

Theoretical:

$$c_2\sqrt{n} \approx 1.73746\sqrt{100,042} \approx 549.55$$

Observed Not PR:

$$541.816$$

Have to double check with new run that does not include 0.

Permutations

Theoretical:

$$0$$

Observed PR:

$$0$$