

# Summarized Results for 100,043 “Safe” Prime

February 13, 2005

## General

**All:** 100,041 Graphs

**Primitive Root:** 50,020 Graphs

**Not Primitive Root:** 50,021 Graphs

**Largest Cycle:** 100,042 ( $g = 20,812$  and  $94,034$ )

**Longest Tail:** 1,448 ( $g = 89,339$ )

**Shortest Maximum Cycle:** 1 ( $g = 72,116$  and  $91,980$  and  $95,997$ )

## Number of Components

**Observed All:**

$$923,855/100,041 \approx 9.23$$

**Maps**

**Theoretical:**

$$\frac{1}{2} \log n = \frac{1}{2} \log 100,043 \approx 5.76$$

**Observed Not PR:**

$$319,587/50,021 \approx 6.39$$

**Permutations**

**Theoretical:**

$$\sum_{i=1}^{100,043} \frac{1}{i} \approx 12.09$$

**Observed PR:**

$$604,268/50,020 \approx 12.08$$

## Number of Cyclic Nodes

Observed All:

$$5,023,873,923/100,041 \approx 50,218.15$$

Maps

Theoretical:

$$\sqrt{\pi n/2} = \sqrt{\pi 100,043/2} \approx 396.418$$

Observed Not PR:

$$19,773,083/50,021 \approx 395.296$$

Permutations

Theoretical:

$$100,042$$

Observed PR:

$$5,004,100,840/50,020 = 100,042$$

## Number of Tail Nodes

Observed All:

$$4,984,527,840/100,041 \approx 49,824.85$$

Maps

Theoretical:

$$n - \sqrt{\pi n/2} = 100,043 - \sqrt{\pi 100,043/2} \approx 99,646.58$$

Observed Not PR:

$$4,984,477,820/50,021 \approx 99,647.70$$

Permutations

Theoretical:

$$1$$

Observed PR:

$$50,020/50,020 = 1$$

## Number of Terminal Nodes

**Observed All:**

$$2,502,250,501/100,041 \approx 25,012.25$$

## Maps

**Theoretical:**

$$e^{-1}n = e^{-1} * 100,043 \approx 36,803.76$$

**Observed Not PR:**

$$2,502,200,481/50,021 = 50,022.99996$$

All values of  $g$  except 100,042 that were not PR had exactly 50,022.  $g = 100,042$  had a total of 100,041. Since  $100,042 \equiv -1 \pmod n$  this exception makes sense. In the other cases, obviously,  $g^0 \equiv g^{p-1} \equiv 1 \pmod n$ . Additionally,  $g^{\text{PR}}$  was always a terminal node. The first two cases are clear. I am likely missing something obvious, but as yet, I do not see why  $g^{\text{PR}} \pmod n$  is always a terminal node.

## Permutations

**Theoretical:**

$$1$$

**Observed PR:**

$$50,020/50,020 = 1$$

## Number of Image Nodes

**Observed All:**

$$100,043 - 25,012.25 \approx 75,030.75$$

## Maps

**Theoretical:**

$$(1 - e^{-1})n = (1 - e^{-1}) * 100,043 \approx 63,239.24$$

**Observed Not PR:**

$$100,043 - 50,022.99996 \approx 50,020.00$$

The notes under Number of Terminal Nodes also apply here since Terminal Nodes+Image Nodes =  $n$ .

**Permutations**

**Theoretical:**

$$n - 1 = 100,043 - 1 = 100,042$$

**Observed PR:**

$$100,043 - 1 = 100,042$$

**Average Tail Length**

**Observed All:**

$$197.95$$

**Maps**

**Theoretical:**

$$\sqrt{\pi n/8} = \sqrt{\pi 100,043/8} \approx 198.21$$

**Observed Not PR:**

$$197.96$$

**Permutations**

**Theoretical:**

$$1$$

**Observed PR:**

$$1$$

## Average Cycle Length

Observed All:

25,089.18

Maps

Theoretical:

$$\sqrt{\pi n/8} = \sqrt{\pi 100,043/8} \approx 198.21$$

Observed Not PR:

198.315

Permutations

Theoretical:

$$\frac{n+1}{2} = \frac{100,043+1}{2} = 50,022$$

Observed PR:

49,980.6

## Maximum Cycle Length

Observed All:

31,321.1

Maps

Theoretical:

$$c_1\sqrt{n} \approx 0.78248\sqrt{100,043} \approx 247.495$$

Observed Not PR:

247.256

## Permutations

Theoretical:

$$0.62432965n = 0.62432965 * 100043 \approx 62,495.81$$

Observed PR:

62,395.5

## Maximum Tail Length

Observed All:

271.41

## Maps

Theoretical:

$$c_2\sqrt{n} \approx 1.73746\sqrt{100,043} \approx 549.55$$

Observed Not PR:

541.816

## Permutations

Theoretical:

1

Observed PR:

1