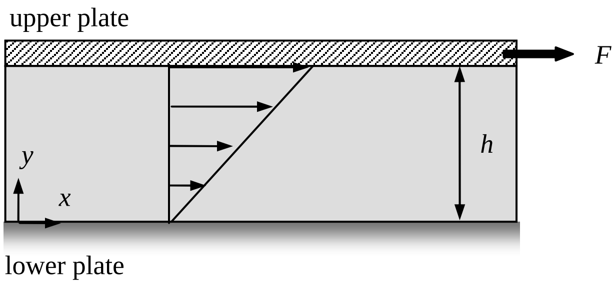
Homework Set #3

A thin layer of fluid separates two horizontal plates (see figure below). The lower plate is stationary and the upper plate moves to the right with a constant speed U. The fluid is a Newtonian fluid with dynamic viscosity μ and density ρ. The surface area of each plate in contact with the fluid layer is A_{plate}.

Because of the system geometry, it is reasonable to assume that the general three-dimensional flow can be simplified using the assumptions shown adjacent to the figure.



1. $V_y = V_z = 0$
2. $V_x = V_x(y)$ not a function of x or z
3. $P = P(y)$ not a function of x or z
4. $g_x = g_z = 0$ and $g_y = -g$

Boundary conditions:

At $y = 0$, $V_x = 0$

At $y = h$, $V_x = U$

a) Show that the following relations govern the flow of the fluid between the plates:

$$\mu \frac{d^2 V_x}{d y^2} = 0 \qquad \text{and} \qquad \frac{d P}{d y} + \rho g = 0$$

[Hint: Start with the three-dimensional Navier-Stokes equation for an incompressible fluid. Use the given information about the velocities, pressure and body force to simplify the equations. Recall that a partial derivative automatically becomes an ordinary derivative when the differentiated variable only depends on one variable.]

b) Use the results of Part (a), and the given boundary conditions to solve for the velocity distribution in the fluid layer. Show the following result is correct:

$$V_x = U \frac{y}{h}$$

c) Use the results of Part (b) to solve for the force F required to move the upper plate at a constant velocity U. Show the following result is correct:

$$F = \tau A_{\text{plate}} = \mu \frac{U}{h} A_{\text{plate}}$$

d) Solve for F/A_{plate} in N/m^2 if U = 1 m/s and h = 5 mm and the fluid is

i. mercury at 20 $^{\circ}$C;

ii. water at 20 $^{\circ}$C;

iii. SAE 10W at 20 $^{\circ}$C;

iv. SAE 50 at 20 $^{\circ}$C.